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MAL-003-001544

Seat No. ________

B. Sc. (CBCS) (Sem. V) Examination

October / November – 2016

Statistics : S-503

(Statistical Inference)

[New Course]

Faculty Code : 003

Subject Code : 001544

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) Q. No. 1 carries 20 marks and Q. No. 2 and Q. No. 3 each carries 25 marks.

(ii) Student can use their own scientific calculator.

1 Filling the blanks and short questions : (Each 1 mark) **20**

(1) A sample constant representing a population parameter is known as _________.

(2) A single value of an estimator for a population parameter θ is called its ________ estimate.

(3) If T_n is an estimator of a parametric function $\tau(\theta)$, the mean square error of T_n is equal to _______.

(4) If $T_n = t_n(X_1, X_2, X_3, \ldots\ldots, X_n)$, an estimator of $\tau(\theta)$, is such that $\lim_{n\to\infty}\left[T_n - \tau(\theta)\right]^2 = 0$, T_n is said to be _______ consistent.

(5) If T_n is an estimator of a parameter θ of the density $f(x;\theta)$ the quantity $E\left[\frac{\partial}{\partial\theta}\log f(x;\theta)\right]^2 = 0$, T_n is said to be ________ consistent.

(6) If $S = s(X_1, X_2, X_3, \ldots\ldots, X_n)$ is a sufficient statistic for θ of density $f(x;\theta)$ and $f(x_i;\theta)$ for $i = 1, 2, 3, \ldots\ldots, n$ can be factorised as $g(s,\theta)h(x)$, then $s(X_1, X_2, X_3, \ldots\ldots, X_n)$ is a ______.

(7) If $f(x;\theta)$ is a family of distributions and $h(x)$ is any statistic such that $E[h(x)] = 0$, then $f(x;\theta)$ is called ______.

(8) If a random sample $x_1, x_2, x_3, \ldots\ldots, x_n$ is drawn from a population $N(\mu, \sigma^2)$, the maximum likelihood estimate of μ is ______.

(9) For a rectangular distribution $\frac{1}{(\beta - \alpha)}$ the maximum likelihood estimates of α and β are ______ and _______ respectively.

(10) Let $x_1, x_2, x_3, \ldots\ldots, x_n$ be a random sample from a density $f(x,\theta) = \theta e^{-\theta x}$. Then the Cramer-Rao lover bound of variance of unbiased estimator is ________.

(11) _________ is an unbiased estimator of p^2 in Binomial distribution.

(12) The estimate of the parameter λ of the exponential distribution $\lambda e^{-\lambda x}$ by the method of moments is ________.

(13) The estimator of σ^2 based on random sample $x_1, x_2, x_3, \ldots\ldots, x_n$ from a population $N\left(\mu, \sigma^2\right)$ by method of moments is _______.

(14) If $x_1, x_2, x_3, \ldots\ldots, x_n$ is a random sample from an infinite population and s^2 is defined as $\frac{\sum\left(x_i - \bar{x}\right)^2}{n}$, $\frac{n}{n-1}s^2$ is an _______ estimator of population variance σ^2.

(15) Let there be a sample of size n from a normal population with mean μ and variance σ^2. The efficiency of median relative to the mean is ________.

(16) Define parameter space.

(17) Name different criteria of good estimators.

(18) Write likelihood function of $f(x, \theta) = \binom{-k}{x} \theta^k (\theta - 1)^x$; $0 \le \theta \le 1$.

(19) Write likelihood function of $f(x, \theta) = \theta(1 - \theta)^{x-1}$.

(20) Obtain Cramer-Rao lower bound of variance of unbiased estimator of parameter of $f(x, \theta) = \theta x^{\theta - 1}$.

2 (a) Write the answer any three : (Each 2 marks) **6**

(1) Define Unbiasedness.

(2) Define Efficiency.

(3) Define Complete family of distribution.

(4) Define Uniformly Most Powerful Test (UMP test).

(5) Define ASN function of SPRT.

(6) Show that $\sum x_i$ is a sufficient estimator of θ for Geometric distribution.

(b) Write the answer any three : (Each 3 marks) **9**

(1) Obtain unbiased estimator of $\frac{kq}{p}$ of Negative Binomial distribution.

(2) $\frac{\bar{x}}{n}$ is a consistent estimator of p for binomial distribution.

(3) Obtain MVUE of parameter θ for poisson distribution.

(4) Obtain estimator of θ by method of moments in the following distribution :

$$f(x;\theta) = \theta x^{\theta-1};\ 0 \le x \le 1.$$

(5) Give a random sample $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ from distribution with p.d.f. $f(x;\theta) = \frac{1}{\theta};\ 0 \le x \le \theta$. Obtain power of the test for testing $H_0 : \theta = 1.5$ against $H_1 : \theta = 2.5$ where $c = \{x;\ x \ge 0.8\}$.

(6) Obtain Operating Characteristic (OC) function of SPRT.

(c) Write the answer any two : (Each 5 marks) **10**

(1) State Crammer-Rao inequality and prove it.

(2) Estimate α and β in the case of Gamma distribution by the method of moments $f(x;\alpha,\beta) = \frac{\alpha^{\beta}}{\Gamma\beta} e^{-\alpha x} x^{\beta-1};\ x \ge 0, \alpha \ge 0$.

(3) Obtain OC function for SPRT of binomial distribution for testing $H_0 : p = p_0$ against $H_1 : p = p_1 (> p_0)$.

(4) Give a random sample $x_1, x_2, x_3, \ldots\ldots, x_n$ from distribution with p.d.f. $f(x;\theta) = \theta e^{-\theta x};\ 0 \le x \le \infty,\ \theta > 0$.

Use the Neyman Pearson Lemma to obtain the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$.

(5) Obtain likelihood ration test :

Let $x_1, x_2, x_3, \ldots\ldots, x_n$ random sample taken from $N(\mu, \sigma^2)$.

To test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \ne \sigma_0^2$.

3 (a) Write the answer any three : (Each 2 marks) **6**

(1) Define Consistency.

(2) Define Sufficiency.

(3) Define Minimum Variance Bound Estimator (MVBE).

(4) Define Most Powerful Test (MP test).

(5) Obtain an unbiased estimator of θ by for the following distribution $f(x;\theta) = \frac{1}{\theta};\ 0 \le x < \theta$.

(6) Show that sample mean is more efficient than sample median for normal distribution.

(b) Write the answer any three : (Each 3 marks) **9**

(1) Let $x_1, x_2, x_3, \ldots\ldots, x_n$ be random sample taken from $N(\mu, \sigma^2)$ then find sufficient estimator of μ and σ^2.

(2) Obtain an unbiased estimator of population mean of χ^2 distribution.

(3) Prove that $E\left(\frac{\partial \log L}{\partial \theta}\right)^2 = -E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right)$.

(4) If A is more efficience than B then prove that $Var(A) + Var(B - A) = Var(B)$.

(5) Use the Neyman Pearson Lemma to obtain the best critical region for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1$ in the case of Poisson distribution with parameter λ.

(6) Let p be the probability that coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 6 times and H_0 is rejected if more than 4 head are obtained. Find the probability of type-I error, type-II error and power of test.

(c) Write the answer any two : (Each 5 marks) **10**

(1) State Neyman-Pearson Lemma and prove it.

(2) Obtain MVBE of σ^2 for normal distribution.

(3) If T_1 and T_2 be two unbiased estimator of θ with variance σ_1^2, σ_2^2 and correlation ρ, what is the best unbiased linear combination of T_1 and T_2 and what is the variance of such a combination ?

(4) For the double Poisson distribution

$$P(X = x) = \frac{1}{2}\frac{e^{-m_1} m_1^x}{x!} + \frac{1}{2}\frac{e^{-m_2} m_2^x}{x!}; 0, 1, 2, \ldots\ldots..$$

Show that the estimator for m_1 and m_2 by the method of moment are $\mu_1' \pm \sqrt{\mu_2' - \mu_1' - (\mu_1')^2}$.

(5) Construct SPRT of Poisson distribution for testing $H_0 : l = l_0$ against $H_1 : \lambda = \lambda_1 (> \lambda_0)$. Also obtain OC function of SPRT.